# A TRIDENT SCHOLAR PROJECT REPORT

NO. 248

DYNAMICS OF DESTRATIFICATION IN THE SEVERN RIVER ESTUARY



## UNITED STATES NAVAL ACADEMY ANNAPOLIS, MARYLAND

This document has been approved for public release and sale; its distribution is unlimited.

20031201 115

## REPORT DOCUMENTATION PAGE

Form Approved OMB No. 074-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including g the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of the collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 7 May 1997	3. REPORT TYPE	AND DATE COVERED	
4. TITLE AND SUBTITLE Dynamics of destratification in the Severn River estuary  6. AUTHOR(S)		5. FUNDING NUMBERS		
Christopher S. Irwin				
7. PERFORMING ORGANIZATION NAME(S)	AND ADDRESS(ES)		8. PERFORMING ORGA	NIZATION REPORT NUMBER
U.S. Naval Academy			USNA Trident Scholar project report	
Annapolis, MD			no. 248 (1997)	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  10. SPONSORING/M		10. SPONSORING/MON	NITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES Accepted by the U.S. Trident Scholar Committee				
12a. DISTRIBUTION/AVAILABILITY STATEME This document has been ap is UNLIMITED.		ease; its di	istribution	12b. DISTRIBUTION CODE
This document has been ap		ease; its di	istribution	
This document has been ap is UNLIMITED.  13. ABSTRACT: As part of an emooring with several oceanog 1995. This mooring had two calinity, temperature, and other top of Michelson Hall. The Severn estuary and determine instruments from September to non-tidal components. Based Severn River estuary falls is criteria. The gradient Richard that once the water column sevent can trigger the overture to the coupling between local was 14. SUBJECT TERMS	oproved for public release.  In the purpose of this project to be purpose of this project to be purpose of this project to becember of 1995 were on the non-tidal salinitiant of the partially-mixed ardson Number was also calculated as a subtidate of the column. Upon wind forcing and subtidate the partial project to be purposed to the partially-mixed ardson Number was also calculated to the partially-mixed ardson of the column. Upon wind forcing and subtidated to the purpose of the column.	odynamic behave byed near the tepths of 2 aneous wind sect was to do ication occur low low pass by and curren (2a) categor alculated for tability induen wind relax of the control of the	vior of the Sevent mouth of the rist. 3 and 4.7 meter reed and directing cument the circular. The time series (LLP) filtered at data, it was dry according to the observation acced by surface contaction, stratifies	rn River estuary, a ver during the autumn of s which took readings of on data were taken from latory pattern of the es provided by the in order to isolate the etermined that the he Hansen and Rattray period. It is concluded cooling, a strong wind d conditions may resume. t a period of 5 days.
This document has been ap is UNLIMITED.  13. ABSTRACT: As part of an emooring with several oceanog 1995. This mooring had two salinity, temperature, and the top of Michelson Hall. It severn estuary and determine instruments from September to non-tidal components. Based Severn River estuary falls is criteria. The gradient Richard that once the water column sevent can trigger the overture to the coupling between local was compared to the severn coupling between local was considered.	oproved for public release.  In the purpose of this project to be purpose of this project to be purpose of this project to becember of 1995 were on the non-tidal salinitiant of the partially-mixed ardson Number was also calculated as a subtidate of the column. Upon wind forcing and subtidate the partial project to be purposed to the partially-mixed ardson Number was also calculated to the partially-mixed ardson of the column. Upon wind forcing and subtidated to the purpose of the column.	odynamic behave byed near the tepths of 2 aneous wind sect was to do ication occur low low pass by and curren (2a) categor alculated for tability induen wind relax of the control of the	vior of the Several mouth of the rimage of the rimage of the rimage of the circument the circument the circument the circument that it was done of the observation could be surface of the circument that it was done to the control of the circument that it was done of the circumen	rn River estuary, a ver during the autumn of s which took readings of on data were taken from latory pattern of the es provided by the in order to isolate the etermined that the he Hansen and Rattray period. It is concluded cooling, a strong wind d conditions may resume. t a period of 5 days.

## DYNAMICS OF DESTRATIFICATION IN THE SEVERN RIVER ESTUARY

by

Midshipman Christopher S. Irwin, Class of 1997
United States Naval Academy
Annapolis, Maryland
(signature)

Certification of Advisors' Approval

Associate Professor Mario E. C. Vieira Department of Oceanography

(signature) 7. 17. 1997 (date)

Professor Reza Malek-Madani Department of Mathematics

(signature)

7 May 97

Acceptance for the Trident Scholar Committee

Professor Joyce E. Shade Chair, Trident Scholar Committee

(day)

#### CHRISTOPHER SMYTH IRWIN

Midshipman First Class United States Navy

Born August 6, 1975 graduated from Noble and Greenough School, Dedham, MA. son of Mr. and Mrs. Paul C. Irwin, Sherborn, MA.

## "Dynamics of Destratification in the Severn River Estuary"

As part of an effort to study the hydrodynamic behavior of the Severn River estuary, a mooring with several oceanographic sensors was deployed near the mouth of the river during the autumn of 1995. This mooring had two current meters located at depths of 2.3 and 4.7 meters which took readings of salinity, temperature, and current velocity. Simultaneous wind speed and direction data were taken from the top of Michelson Hall.

The purpose of this project was to document the circulatory pattern of the Severn estuary and determine how the fall destratification occurs. The time series provided by the instruments from September to December of 1995 were low low pass (LLP) filtered in order to isolate the non-tidal components. Based on the non-tidal salinity and current data, it was determined that the Severn River estuary falls into the partially-mixed (2a) category according to the Hansen and Rattray criteria. The gradient Richardson Number was also calculated for the observation period.

It is concluded that once the water column shows very low dynamic stability induced by surface cooling, a strong wind event can trigger the overturning of the column. Upon wind relaxation, stratified conditions may resume. The coupling between local wind forcing and subtidal flow was most significant at a period of 5 days.

#### **FACULTY ADVISORS**

Associate Professor Mario E. C. Vieira

Oceanography Department

B.S., Portuguese Naval Academy

M.S., U.S. Naval Postgraduate School

M.A., Ph.D., The Johns Hopkins University

Professor Reza Malek-Madani

Mathematics Department

B.S., M.S., Southern Illinois University

Ph.D., Brown University

keywords: circulation, destratification, estuary, Richardson Number, Severn River, tributary

#### **PREFACE**

In recent years, the field of estuarine research has become not only important, but increasingly diverse. Studies now range from the physical and chemical areas to the biological. This project represents just one subset of this ever-growing field. It also includes the culmination of much more than just my own work. Long before my involvement began, the Environmental Protection Agency (EPA) provided the buoy and the CTD that were used to record data. The interest of such organizations in estuarine oceanography has made this investigation and others like it possible.

Many people are not only interested, but concerned about how the circulation in the Severn River estuary occurs and its ecological effects. For instance, a lack of oxygen being circulated to the bottom can cause huge reductions in benthic organisms such as clams and crabs. I would like to begin by thanking these people and others who support and perpetuate the effort to continue studying estuaries. I had the chance to meet many of these people recently at the Atlantic Estuarine Research Society (AERS) 1997 spring conference. They made me feel at home in their community and built my confidence in my own work. I appreciate all of their support.

In closer connection to this project, there have been many people who have had the wisdom and patience to guide me in my efforts. I would like to thank first and foremost my advisors, Associate Professor Mario Vieira and Professor Reza Malek-Madani. Without their constant care, supervision, and advice, this project would never have been successful.

Professor Vieira began work with this mooring over three years ago. His knowledge and love of oceanography have been an inspiration to me throughout this past year. Likewise, Professor Malek-Madani has spent the last eight months guiding me through the language of Mathematica on the computer. His ability to verse me in the use of this tool, while simultaneously progressing with the project analysis has been vital to its completion. Without the help of both of these men, none of this would have been possible.

I would also like to thank my family. Although I'm quite sure they have very little knowledge about oceanography and probably less interest in the subject, their support and belief in my ability to excel in all that I do has been essential to my success thus far. Without the backing of my parents, I would definitely not be where I am today. I owe them a great deal. Thanks also go to Professor Shade and the rest of the Trident Committee, all of my professors and coaches here at the Naval Academy, and everyone else that has touched my life throughout my college career.

## TABLE OF CONTENTS

Al	Abstract	
Pr	Preface	2
1.	. Introduction	5
2.	2. Background	5
3.	3. Data	8
	3.1 Water Properties	8
	3.2 Wind	9
	3.3 Data Reduction	9
	3.4 Mathematical Preliminaries	
4.	. Results	11
	4.1 Classification	
	4.3 Current Response	
	4.5 Wind Stress	
	4.6 Energy	
	4 = 5, 10	
5.	5. Conclusions	21
6.	5. Recommendations	21
7.	. Naval Applications	22

			4
Notes		23	
Re	ferences		24
A.	List of	Figures	26
B.	B. Mathematica Programs		27
	<b>B</b> . 1	Calculating the Lanczos Filter	27
		Filtering and Plotting Parameters	<b>28</b> .
	B.3	Calculating the Longitudinal Component of the Current or Wind	28
		Calculating Fourier Coefficients	28
		Calculating the Energy Density Spectrum	29
		Calculating Coherence	29
		Calculating the Gradient Richardson Number	30

#### 1. Introduction

The classic two-layered estuarine circulation model was developed by Pritchard (1952, 1954, 1956), assuming steady-state conditions in the subtidal circulation and no surface stress due to the wind<sup>1</sup>. Following studies, however, determined that subtidal circulation varies both temporally and spatially. An estuary does not necessarily have two layers all the time and the wind stress is not negligible.

A study done in the Pamlico River Estuary, NC, by Stanley and Nixon (1992) showed that stratification events are directly related to variations in freshwater discharge and wind stress. A two-dimensional flow model was developed by Wang and McCutcheon (1993) involving tide and density parameters for an estuary-river model. A separate model for mixing was proposed by Geyer and Smith (1987) involving shear flow in a highly stratified salt wedge estuary. The conclusion was that various factors determine the response of an estuary, as the relative energies of the driving forces change all the time.

Wind can have a serious effect on tidal and gravitational circulation (Weisberg 1976). The works of Elliott and Hendrix (1976) and Wang and Elliott (1978) show strong correlations between meteorological forcing and subtidal circulation. Small tributaries, such as the Severn, are prime places to study such occurrences. They provide easier studies than large estuaries and are of great interest when studying local meteorological effects.

In this sense, the Severn River estuary provides a perfect laboratory environment. Like so many other estuaries in this country, it is under enormous ecological stress resulting from urbanization, poor water quality, and turbidity. However, there are no recorded studies of the hydrodynamic behavior of the Severn River estuary, the first step to an understanding of its functioning.

This investigation seeks to fill this gap. It provides the first classification of the Severn River estuary, using the Hansen and Rattray method. More importantly, it examines, for the first time, the meteorologically-driven circulation of a tributary estuary in the Chesapeake Bay. Utilizing time series analysis and a calculation of the Gradient Richardson Number, this project specifies the factors which cause the fall destratification in the Severn River estuary. This study also opens the possibility of directionally dependent wind-driven circulation: that destratification may require wind stress in a specific direction.

#### 2. Background

An estuary is defined as "a semi-enclosed coastal body of water which has a free connection with the open sea and within which sea water is measurably diluted with fresh water from land drainage" (Pritchard, 1967). The amount of mixing that occurs between these two bodies of water varies from place to place, and results in different types of estuaries. The Chesapeake Bay estuary, for instance, is classified as a partially mixed estuary, categorized by fresh river water flowing into a saltwater bay which has a

moderate tidal range. Where these two water masses meet, mixing occurs as denser sea water flows in underneath the less dense river water flowing out. The degree of mixing resulting from this process depends upon the intensity of the tides, the season, the prevailing conditions of freshwater inflow, and the meteorological activity at that time. The amount of vertical mixing determines the stratification of the water column at any particular point in time and space.

Stratification is the degree to which the water column is separated into density layers. Normally, fresher (lighter) water overlays saltier (heavier) water. In an estuary, salinity determines density of the water to a much greater extent than the temperature of the water. Density and tidal mixing determine the structure of the column. However, other variables such as subtidal currents, wind, temperature changes, and freshwater inflow rates all play important roles in changing stratification (Schubel, 1971). Based on previous studies, and given the moderate degree of mixing in the Chesapeake Bay, a mildly stratified density profile typically exists. There are significant amounts of both runoff from the land and heating of the surface layer by the sun. However, the exact profile varies depending upon the time and place.

The structure of the water column is not a constant; it changes throughout the seasons. During the summer, the water is strongly stratified in distinct, discernable density layers from the surface to the bottom. At this time, the halocline, thermocline, and pycnocline are well established and defined<sup>2</sup>. Aside from large density differences resulting from the spring freshet, other factors contribute to this result.

Solar radiation does not penetrate to great depth and only heats the surface layer of water. This increase in temperature results in a decrease of the surface water density. This process increases the stability of the water column and hinders mixing throughout its depth. The summer months also typically show a marked decrease in wind activity, resulting in less wave action which aggravates the deficit of energy available for mixing through the pycnocline.

In the summer, dissolved oxygen concentrations decrease going down through the column because of the increased water temperature at the surface. This situation is exacerbated due to runoff from the land being heavily loaded with nitrogen and phosphorus from agricultural fertilizers and urban effluents. These nutrients enter the water column and stimulate phytoplankton growth aided by the increased light intensity. This in turn supports a large zooplankton and nekton population. As these organisms die, they are broken down on the bottom by bacteria. This activity requires oxygen. With increased organisms, more oxygen is used by the benthic bacteria. This process typically leads to bottom hypoxia or even anoxia<sup>3</sup>.

Other factors such as precipitation also affect stratification in the bay. Increased precipitation will add to the fresh, warm water on top of the column. This will strengthen the pycnocline and increase the difficulty of circulating oxygen to the bottom.

This stratified environment tends to weaken as the season progresses into the autumn and winter. However, exactly how this process occurs is unclear. It is evident that as the winter approaches there are increased winds and decreased solar heating. The amount of correlation that exists between these factors is not apparent. It is also unclear at what rate this destratification occurs, all at once or as a long slow process.

A study was performed by Goodrich (1987) examining the effects of wind on destratification in the Chesapeake Bay. He used surface and bottom measurements taken from five current meter arrays placed in the main body of the bay during the autumn of 1981. These instruments measured current velocity, temperature, and conductivity. His observations proved that much of the autumn destratification that occurs in the Chesapeake Bay is due to wind and storm-related events. Goodrich also observed that the overlying air temperature played a significant but secondary role in helping the breakdown of the water column. Large internal velocity shear seemed to precede significant mixing events. Additionally, the wind effects were depth dependent in stratified conditions and depth-independent in mixed states.

Given the smaller dimensions of a tributary, such as the Severn, there is a question as to how differently it behaves from the main bay. This study attempts to elucidate how the destratification in the Severn Estuary is initiated in the autumn. The investigation involved a cooperative agreement between the U.S. Naval Academy and the Environmental Protection Agency. Technical and human resources of both institutions were pooled to deploy, maintain, monitor, and recover an oceanographic mooring in the Severn River for the purpose of acquiring data to support this study.

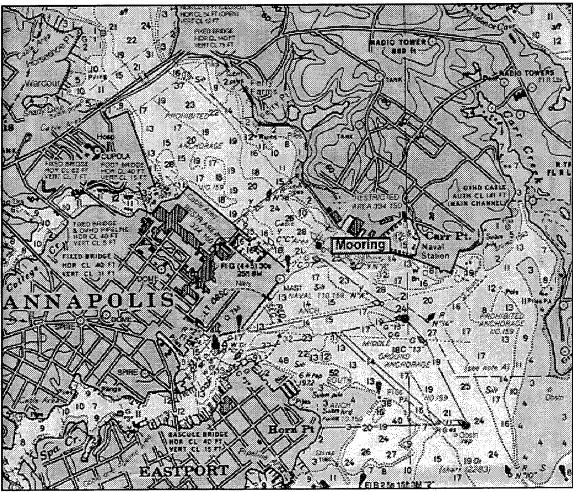


Figure 1. 1995 location of the mooring.

#### 3. The Data

#### 3.1 Water Properties

This project involved data recovered from a mooring deployed at the mouth of the Severn River from 1 September to 7 December 1995. A surface buoy was anchored to the bottom of the river at 38°58.98'N, 76°28.43'W in 8 meters of depth. The location of the mooring is shown in Figure 1. Two S4 InterOcean electromagnetic current meters were placed at depths of 2.3 meters and 4.7 meters from the surface. These meters were representative of the top and bottom halves of the water column respectively. They measured current velocity, temperature, and conductivity every twenty minutes during the three months of the deployment. Two orthogonal components of the current were measured by two pairs of electrodes located at right angles to each other. As water flowed past these sensors, a voltage potential difference was generated proportional to the current velocity. A CTD also was placed at 3.5 meters depth in order to record dissolved oxygen concentrations. A cross section of the mouth of the river and a graphic of the mooring is shown in Figure 2.

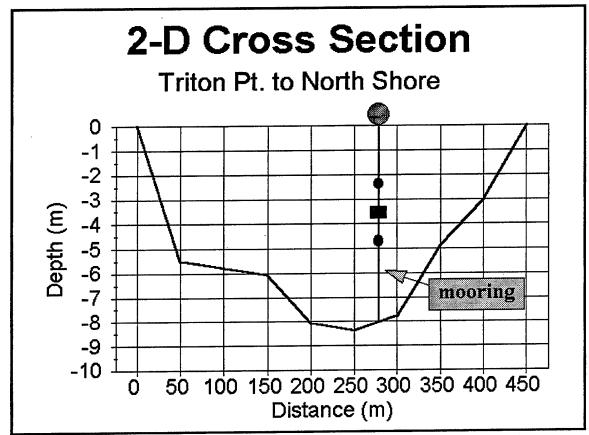


Figure 2. Vertical cross section of the mouth of the Severn River with the mooring deployed, showing the locations of the S4 InterOcean current meters (•) and the CTD (•).

The salinity was calculated from the conductivity and temperature through the equation of Practical Salinity Scale 1978. Density was computed from the salinity and temperature data using the Equation of State of Seawater 1980<sup>4</sup>. There is no information available on the freshwater inflow into the Severn River. Because of the small volumes involved, this waterway has never been gauged.

#### 3.2 *Wind*

Wind speed, direction, and air temperature readings were taken at the top of Michelson Hall (on the grounds of the U.S. Naval Academy) during the same time. Although these measurements were not made at the same location as the mooring, they are expected to give a good representation of local meteorological activity at the time. Readings of wind direction and velocity were recorded every hour. However, 36° had to be added to the recorded wind direction due to an error in the calibration of the direction sensor. Wind speeds were converted to stress using the quadratic law with a drag coefficient of  $1.5 \times 10^{-3}$  (Pond, 1975).

#### 3.3 Data Reduction

The data were reduced and processed through the use of the Mathematica program available on the CADIG computer system of the Division of Engineering and Weapons at the U.S. Naval Academy. Initially the raw data sets for temperature, salinity, density, current, air temperature and wind velocity were uploaded, plotted, and scanned. These sets were filtered using a Lanczos low low pass (LLP) filter with a 34 hour cutoff period in order to remove the astronomical tidal components and isolate the subtidal components of each variable following the method of Vieira (1985). This filter was preferred because of its avoidance of the Gibb's Phenomenon (Bloomfield, 1976), an overshoot of the response on either side of the cut-off frequency. One example of this filtering result on current readings taken by the top meter can be seen in Figure 3.

In order to focus on the portion of the wind and current most prominent in the flow of an estuary, the analysis focused on the longitudinal component of the current and wind vectors. In a similar study, Weisberg (1976) clearly correlated the bottom flow of the Providence River to the local longitudinal wind. For the purposes of this study, the longitudinal component was determined as that running from 310°T to 130°T. The upriver direction was labelled as positive and the downriver direction was labelled as negative.

#### 3.4 Mathematical Preliminaries

The Mathematica programs used in this project first had to be written before any steps involved in the actual analysis could be performed. The beginning of this process involved uploading the data sets into the computer. These were very large lists of numbers, some adding up to approximately 7,000 readings. They were also in various formats and had to be converted to one that Mathematica could recognize and manipulate.

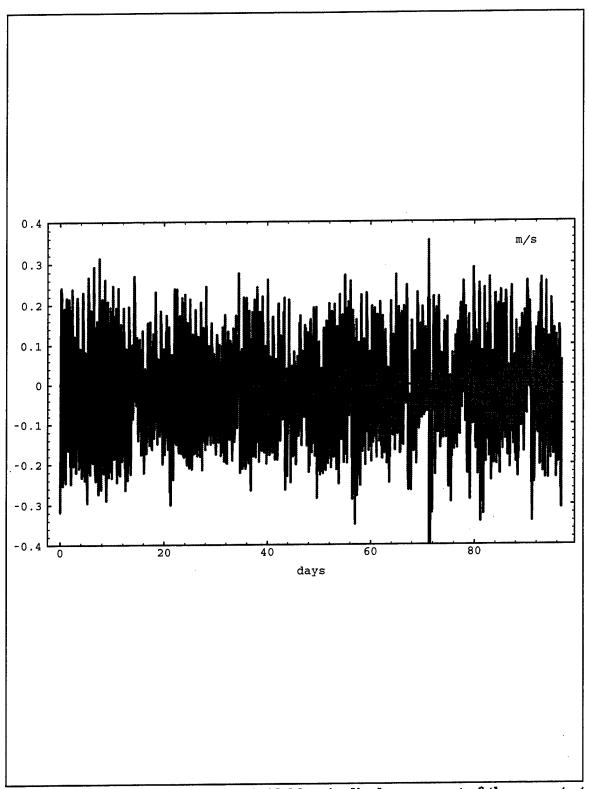


Figure 3. 1995 raw and filtered subtidal longitudinal component of the current at 2.3 meters depth.

This part of the project required a fair amount of study to become familiar with the methods of spectral analysis.

The Lanczos filter was computed first (see Appendix B.1). This filter took the first points of the raw data sets that were the same length as the filter and multiplied them each by a certain weight. The filter was then stepped down the length of the set and this process was repeated.

The wind stress and current required not only unit conversion, but a breakdown into its component vectors. The computation of the longitudinal component required that each direction be matched with its velocity and multiplied by a matrix which shifted the axis used to measure direction by 40° in order to get this component (see Appendix B.3).

Calculating the Fourier coefficients of the filtered data sets provided a particular dilemma. Mathematica includes a "Fourier" command which automatically calculates the Fourier coefficients of any list of numbers. However, it divides each coefficient by a constant equal to the square root of the length of the list. Therefore, a more drawn-out calculation was used that divides by merely the length of the data set. This process required rewriting the programming to compute coefficients which provided better results. Mathematica also calculates a value for the mean which appears as a very strong peak at the beginning of the power series. By demeaning the filtered sets and calculating the spectrum through original programming, this huge peak was avoided (see Appendix B.4).

These energy spectra had to be smoothed in order to see correlations between significant frequencies. This smoothing was performed by taking a certain number of points, adding their values, and dividing by the total number of points. This provided one average value in place of the original numbers. Unlike the filtering, this window was moved the entire length of the averaging factor. Many averaging factors were tried and 11 points was settled upon as the best. Factors from 7 up to 18 points provided the same general peaks but 11 gave the best representation of the strongest frequency (see Appendix B.5).

#### 4. Results

#### 4.1 Classification

The Hansen-Rattray method was used to classify the Severn estuary (Hansen and Rattray, 1966). This is a quantitative method which only requires the values of the salinity and current velocity in order to categorize an estuary from the hydrodynamic point of view. Essentially it is a determination of the ratio of the stratification parameter to the circulation parameter. The stratification parameter shows the difference between the surface and bottom salinity divided by the mean cross sectional salinity. The circulation parameter expresses a ratio of the surface flow (including that mixed in by entrainment or eddy diffusion) to the mean cross-sectional flow. The data for this classification were obtained by creating a cross section of the mouth of the Severn River (shown previously in Figure 2) using interpolated values of depth from chart 12283 (1995 version). Utilizing a grid, and considering 3.5 meter depth as the level of no motion

between the two sensors, a two-dimensional area was calculated for the top and bottom layers. The top and bottom sensors were then assumed to represent these two layers respectively.

This point appears as a dot on the Hansen-Rattray diagram (Figure 4). The variables in the ordinate are:  $\bar{S}_b$ , the average bottom layer salinity,  $\bar{S}_s$ , the average top layer salinity, and  $<\bar{S}>$ , the average salinity throughout the entire column. The variables in the abscissa are:  $\bar{U}_s$ , the average velocity in the surface layer, and  $\bar{U}_x$ , the average cross sectional velocity through the column. An (a) or (b) indicates the amount of stratification which occurs; Type (a) being slightly stratified and Type (b) showing considerable stratification. Type 1 has a net flow seaward and diffusion controls salt transport upstream describing a sectionally mixed estuary. Type 2 is a partially mixed estuary where subtidal flow at depth is opposite that in the upper layer. Salt flux upstream is controlled by both diffusion and advection in this case. Advection controls most of the salt transfer in Type 3. Type 4 is a salt wedge.

The Severn River estuary was classified as a partially mixed estuary with very little stratification (2a). This is shown in Figure 4. The small change in density between the top and bottom layers is resultant of the moderate tidal range in the Severn River which provides a large degree of mixing compared to the small amount of fresh water inflow coming into the estuary from Severn River runoff and smaller tributaries.

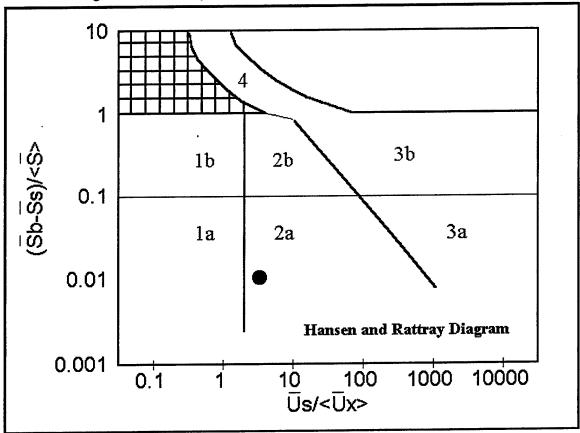


Figure 4. Hansen and Rattray diagram showing the classification of the Severn River estuary (•) - Type (2a), a partially mixed estuary with little stratification.

For most estuaries, this classification does not always remain constant and may vary slightly with different runoff regimes. The separation into different types is also somewhat arbitrary, as it is difficult to place a label on any given estuary. For the purposes given here, the Hansen-Rattray method proved adequate. Indeed, this is probably the most satisfactory classification system yet proposed<sup>5</sup>.

#### 4.2 Air and Water

The water responds to changes in the air temperature. As the air cools from 1 September to 7 December of 1995 (Figure 5(a)), the water temperature slowly drops (Figure 5(b)). However, this decrease is not as great in magnitude as that of the atmosphere. The temperature of the water diminishes at a much slower rate. This difference is due to the high heat capacity of water. A greater degree of energy is required to change the temperature of a volume of water compared to an equal volume of air.

This results in a slight thermal inertial delay, which is shown in Figure 5. The air temperature fluctuates but slowly drops over the three month period. This can be seen clearly at the point in time labelled (1) where there is a sharp decrease in the air temperature and the water follows this trend to a lesser degree.

Figure 5(b) illustrates another significant point concerning the water column. The temperature difference between the top and bottom layers is relatively slight. At the beginning of the season, the surface layer is slightly warmer than the bottom layer due to solar radiation from the summer. At approximately the same point that the air temperature begins to decrease, the temperature of the two layers becomes equal. The significance of this occurrence is shown further in Figure 6(a). In this plot, positive values indicate warmer temperatures in the top layer; this is a stabilizing factor. From day 36 on, the upper layer is almost always cooler than the lower layer; this is a destabilizing condition. But this drop in temperature alone is not enough to overturn the water column. Other factors must be considered.

A more important change is seen in the salinity difference between the two layers. In Figure 6(b) positive numbers indicate more saline water in the top layer *i.e.* a destabilizing condition. In the first 14 days, the salinity of the upper layer is lower than that of the bottom, as expected from classical estuarine circulation. After this point, the upper layer becomes more saline until day 76 or so when again the top layer becomes fresher.

As a result, the temperature reinforced the salinity in creating a stable condition during the first 14 days, after which the column became unstable until day 76. This is clearly shown in Figure 6(c). This plot shows the density differences clearly being controlled by the salinity. After day 76, in spite of increased cooling of the top layer, the column becomes stable again due to the suddenly restored fresher water on top.

This phenomenon is referred to as an inversion. The water column becomes topheavy and unstable. As a result, the water column destratifies and mixes. Due to the slight difference in temperature and the large difference between salinities, it is clear that the salinity is more of a driving factor in creating instability. The temperature is

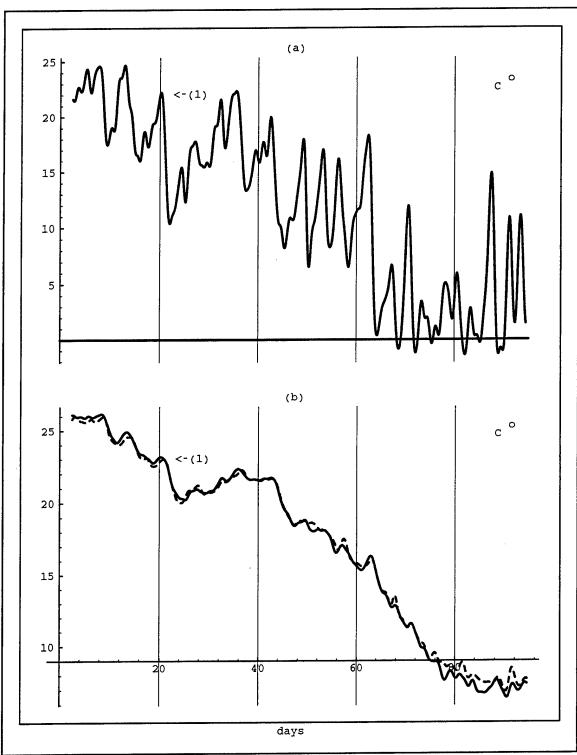


Figure 5. 1995 (LLP) filtered air temperature (a) and water temperature (b) time series. Water temperature is at depths of 2.3 meters (solid line) and 4.7 meters (dashed line). Point (1) shows the response of the water temperature to that of the air.

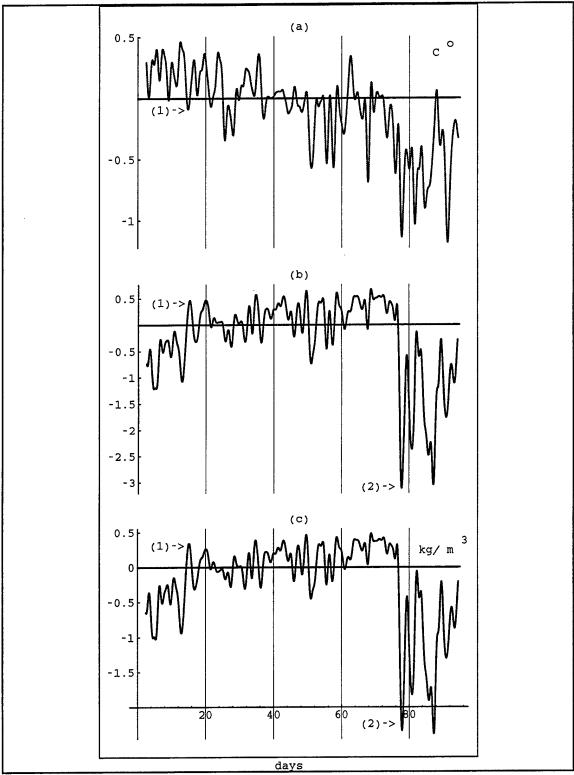


Figure 6. 1995 (LLP) filtered differences from 2.3 meters to 4.7 meters depth (top - bottom), (a) temperature, (b) salinity, (c) density. Point (1) indicates the inversion. Point (2) shows the restratification.

important but secondary to the salinity in determining when this inversion will take place. However, the column still needs a force to initiate this change. This is found in the currents and the wind.

#### 4.3 Current Response

The LLP filtered currents are shown in Figure 7. As stated before, a positive flow is upriver and a negative flow is downriver (into the Chesapeake Bay). At the beginning of the season, the column behaves as would be expected in a classical estuarine flow. The average flow for the top layer flow is negative and the average flow of the bottom layer is positive. This corresponds to the top, fresher river water flowing out over the bottom, saltier bay water.

The flow of the two layers is decoupled throughout the first 58 days or so. The peaks of bottom and top flow do not line up with each other on the plot. They are actually out of phase by 180°. After the inversion of the water column however, this trend begins to disappear. The flows become coupled after day 58 and fluctuate together.

This change in flow is due to the mixing of the water column. As the layers overturn, the density gradient disappears and the response of the column to changes in the atmosphere is not a function of the depth. This remains true until the end of the season when the column begins to restratify and the layers decouple again.

#### 4.4 Richardson Number

Until this point, the stability of the water column has been discussed and related to fluctuations of certain parameters. The difference in temperature, salinity, and the resultant density have been used to show the instability of the water column past a certain point in the season. However, this condition must now be quantified. The dynamic stability of a stratified column of water away from boundaries can be quantified through the gradient Richardson number, Ri (Turner, 1973):

$$Ri = \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} / (\frac{\partial u}{\partial z})^2$$

The Richardson number provides a unitless numerical value of the comparison of stratification and destratification forces, where g is the acceleration due to gravity,  $\rho_0$  is a reference density,  $\partial \rho/\partial z$  is the vertical density gradient, and  $\partial u/\partial z$  is the vertical gradient of longitudinal velocity or shear (Goodrich *et al.*, 1987). The ratio of these two vertical gradients shows the relationship between the two opposing forces: density which tends to stratify, and shear which tends to mix. Negative values represent an unstable situation, and positive values, a stable situation. Intensity in either direction increases with magnitude. In this study, all low low pass filtered values were used to calculate this number; the reference density was provided by an average of all the filtered densities for

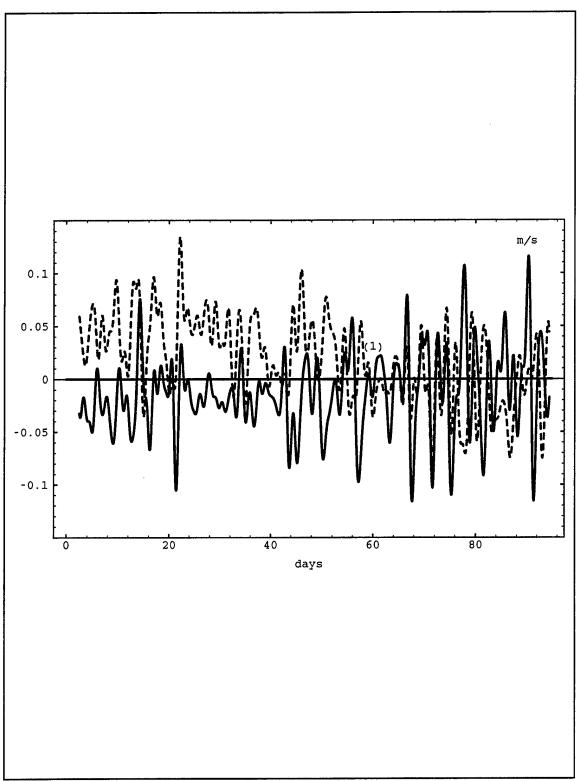


Figure 7. 1995 (LLP) filtered subtidal longitudinal component of the current at 2.3 meters (solid line) and 4.7 meters (dashed line) depth. Point (1) indicates the coupling point.

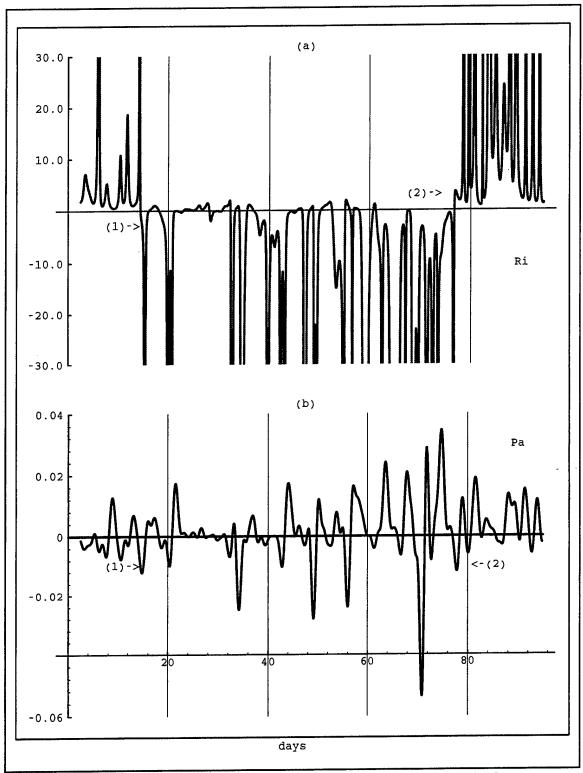


Figure 8. 1995 Gradient Richardson Number (a) and the longitudinal component of the wind stress (b). Point (1) indicates the inversion. Point (2) shows the restratification.

the three-month set.

The Richardson Number for this data set is shown in Figure 8(a). The scale has been arbitrarily set from 0 to 30 as larger values of stability and instability increase in magnitude towards infinity. A very clear overturning of the water column is evident on day 14 where the numbers shift from very high positive to very low negative. Towards the end of the season, near the middle of November, this process reverses and the column restratifies.

#### 4.5 Wind Stress

The effect of the wind on the water column is apparent in this process. The longitudinal component of the wind stress is shown in Figure 8(b). At the same time as the inversion (day 14), a significant down estuary wind event is apparent. This wind force causes friction on the top of the water column and stress in the same direction of the upper layer flow, *i.e.* out of the estuary.

This effect is seen in the current plot in Figure 7. The current increases dramatically and the first evidence of coupling is seen. This is the same point as that identified for destratification. After this point in time, there is a period of fairly low wind stress and the water column is mostly close to neutral stability. Past this, just beyond day 42, the wind increases for the rest of the season and the column becomes very unstable until day 76.

This instability and mixing are dependent upon the wind and sustained wind conditions throughout the middle of the autumn. More specifically, down estuary winds seem to encourage the overturning of the water column. This is seen in similar peaks between the wind stress and the Richardson Number. Large peaks of wind stress in the negative direction match up with peaks of large negative values in the Richardson Number. Peaks in the positive direction tend to bring the column back towards stability.

At the end of the season, after day 76, the Richardson Number indicates a restratification of the water column. This occurs concomitantly with a significant wind event going upriver. After that the stress remains largely in the positive direction and with reduced magnitude. This occurrence provides strong evidence that not only the magnitude but the direction of the wind stress have a significant effect on water column stability.

#### 4.6 Energy

Figure 9 shows the plots of the energy density spectra for the top layer current and the wind stress. As seen in both 9(a) and 9(b), there is a strong frequency of importance at approximately 0.2 cycles per day. This corresponds to a period of approximately 5 days, considered to be a representative time for a weather system to move through the area. As seen before, the water responds to the air. This significant period of 5 days is another evidence of the coupling between local wind forcing and subtidal water flow.

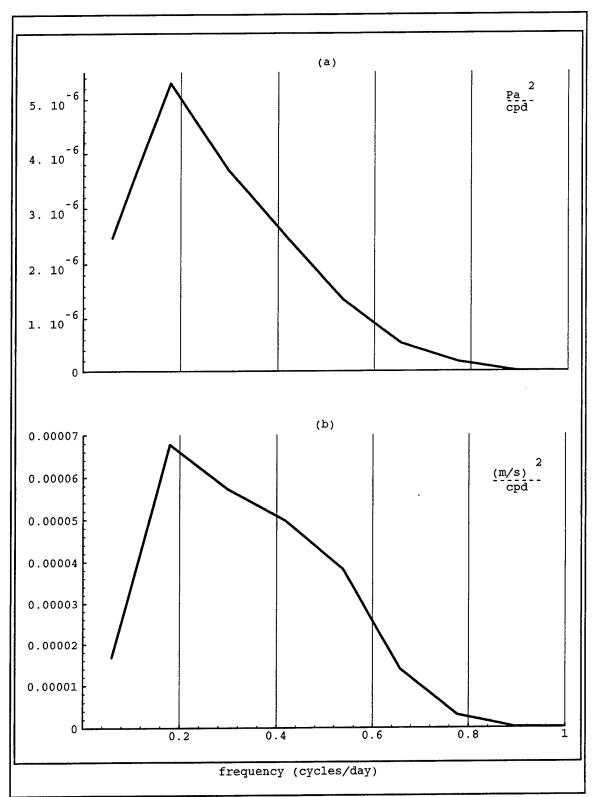


Figure 9. 1995 energy density spectra of the (LLP) filtered longitudinal components of the wind stress (a) and the current at 2.3 meters depth (b).

#### 4.7 Dissolved Oxygen

Oxygen data, retrieved from the CTD at about 3.5 meters depth, was reduced and filtered similarly to the other data. However, questions arose as to the validity of the readings. The instrument itself had experienced problems during the recording period. As a result, the filtered values did not show any conclusive findings regarding dissolved oxygen concentrations. Accordingly, no effort will be made to utilize these data further.

#### 5. Conclusions

It is clear that the meteorology of the Chesapeake Bay area affects the Severn River estuary. The process of destratification begins with lowering of the surface layer temperature by the decreasing air temperature. As a result, the temperature gradient decreases and the salinities between the top and bottom layers begin to equalize. At this stage, the water column is not mixed, but holds the capacity to destratify. However, this process requires another energy source. The air temperature is definitely a factor in lowering the stability of the column, but it cannot stand on its own. Merely lowering the temperature will not change the structure of the water.

The trigger is provided by the wind. The wind does affect the water column and is the final factor in the destratification process. As the wind blows, frictional processes create turbulence in the water. If the stability of the column is already very low, a strong wind event may be all that is necessary to start the overturn of the water. The response is not only velocity dependent, but directionally dependent. The winds whose fetch is maximized in the direction of the waterway will have the most effect on the mixing of the layer.

A period of 5 days was seen to be significant in both the longitudinal wind stress and the top current; this may coincide with the passage of weather systems in this area and is consistent with other determinations of local wind forcing effects (e.g. Vieira, 1986).

#### 6. Recommendations

Correlation of the wind stress and the current was never successfully performed on this data. This calculation is called the coherence and the Mathematica program which was created to perform this calculation is in Appendix B.6 of this report. This program, however, never gave satisfactory results. Coherence should have a value between 0 and 1. This was not always the case in the calculations made.

In addition, the possibility of the fall destratification being dependent not only on the strength, but also on the direction of the wind stress is suggested. This study does not offer conclusive evidence that this is the case, but the theory has never been explored and the implications seen in this examination certainly warrant future work. The almost identical peaks of instability in the Gradient Richardson Number and the negative

(downriver) values in the wind stress seem to be more than mere coincidence. Similarly, positive (upriver) values of wind stress line-up with periods of restabilization. This theory should be explored further in future studies. An experiment could certainly be specifically designed to help prove or disprove this suggestion.

### 7. Naval Applications

Understanding the circulation in coastal waters has always been important for amphibious operations. Amphibious landings such as those by U.S. Marines in the Second World War were heavily dependent upon tidal predictions. Operations such as the landing at Tarawa suffered heavy casualties due to incorrect tidal predictions. This problem was seen in the Korean War as well.

With an increasing focus on littoral warfare, the need for understanding the dynamics of coastal waters and riverine environments has become increasingly important. This became apparent during the Vietnam Conflict when this type of warfare was first necessary. Since then, regional conflicts spanning the globe have necessitated, to a much greater extent than ever before, the knowledge of coastal and interior water structures.

In modern times, covert amphibious operations have become increasingly common. Tides are still a factor, but understanding the structure of the water column and how its make-up changes with the season has become necessary as well. Although submarines cannot penetrate deep into river environments, special forces groups such as Navy SEAL teams use mini-submersibles that are designed specifically for covert underwater operations in shallow water areas. Information such as that studied in this project could be vital for operations involving Navy divers who must travel through this environment in order to accomplish a mission.

These manned mini-submarines include the currently used SEAL Delivery Vehicle (SDV) and the new Advanced SEAL Delivery System (ASDS). The SDV is a mini-submersible which is deployed from a submarine and carries up to two dive pairs in a water-filled hull. Inside this vessel, the divers breath using scuba gear. The ASDS is a dry vehicle on the inside and can transport up to eight SEALs.

These vessels are highly dependent upon buoyancy to maintain cover and concealment. Therefore, the depth of one of these submersibles is directly related to the density of the waters. Sudden changes in water density can cause one of these submersibles to experience an uncommanded ascent, thereby alerting an enemy to the presence of the divers and compromising the mission.

Therefore an understanding of the structure of the water column is vital to mission success. This includes knowledge about the air as well as the water. Specifically, understanding how meteorology affects the water column and how circulation will change with the winds and temperature shifts could lead to success in a mission. Not comprehending the affects could result in failure.

Studies such as this one could add to the understanding of how water columns around the world react to meteorological changes. Estuaries exist in all corners of the globe and an in-depth knowledge of how different types of estuaries change with the

seasons would certainly aid special operations. A mission plan that included the make-up of the water column could be vital to mission success.

#### NOTES

- 1. Much of the flow in an estuary is due to the tides, a very regular and predictable occurrence. Subtidal circulation refers to that not attributed to the tides.
- 2. Factors such as salinity, temperature, and density vary throughout the water column. Typically an area exists where each factor changes rapidly within a short distance. Within the halocline, the salinity usually shows a marked increase. The pycnocline contains a similar occurrence for density. The thermocline is an area of sharp temperature decrease.
- 3. Hypoxia and anoxia are cases where the amount of dissolved oxygen within the water is dramatically reduced. Hypoxic conditions still maintain a minimal amount of oxygen. Anoxic conditions contain no oxygen.
- 4. Although these two equations were used to perform the conversions, these steps were executed before this project began.
- 5. It is evident that this method assumes a two-layer estuarine model with each sensor representative of a layer of water. As seen in previous studies, this may not always be the case. Additionally, the placement of the meters involved in this experiment may be considered somewhat arbitrary. However, they were deployed before this study began and their positions were not a variable factor. Therefore, the Hansen and Rattray method is considered the best way to classify this estuary because the criteria requires only two layers and utilizes their salinity and current measurements.

#### REFERENCES

- Bendat, J. S., & Piersol, A. G. (1971). <u>Random Data: Analysis and Measurement Procedures</u>, Wiley-Interscience.
- Bloomfield, P. (1976). <u>Fourier Analysis of Time Series: An Introduction</u>, John Wiley & Sons.
- Brush, G. S. (1991). Stratigraphic History Helps Us Understand Today's Chesapeake Bay. Geotimes, 36(12), 21-23.
- Elliott, A. J., & Hendrix, T. E. (1976). Intensive Observations of the Circulation in the Potomac Estuary. Chesapeake Bay Institute, The Johns Hopkins University, Spec. Rept. #55, 35 pp.
- Geyer, W. R., & Smith, J. D. (1987). Shear Instability in a Highly Stratified Estuary. Journal of Physical Oceanography, 17, 1668-1679.
- Goodrich, D. M., Boicourt, W. C., Hamilton, P., & Pritchard, D. W. (1987). Wind-Induced Destratification in Chesapeake Bay. <u>Journal of Physical Oceanography</u>, 17, 2232-2240.
- Hansen, D. V., & Rattray, M. (1966). New Dimensions in Estuary Classification. <u>Limnology and Oceanography</u>, <u>11(3)</u>, 319-326.
- Pond, S. (1975). The Exchange of Momentum, Heat, and Moisture at the Ocean-Atmosphere Interface. <u>Numerical Models of Ocean Circulation</u>, Natl. Acad. Sci., Washington, DC, 26-38.
- Pritchard, D. W. (1952). Salinity Distribution and Circulation in the Chesapeake Bay. Journal of Marine Research, 11, 106-123.
- Pritchard, D. W. (1954). A Study of the Salt Balance in a Coastal Plain Estuary. Journal of Marine Research, 13, 133-144.
- Pritchard, D. W. (1956). The Dynamic Structure of a Coastal Plain Estuary. <u>Journal of Marine Research</u>, <u>15</u>, 33-42.
- Pritchard, D. W. (1967). What is an Estuary: Physical Viewpoint, Estuaries, 83, 757p.
- Pritchard, D. W. (1978). What have Recent Observations Obtained for Adjustment and Verification of Numerical Models Revealed about the Dynamics and Kinematics of Estuaries? <u>Estuarine Transport Processes</u>, (Kjerfve, B., ed.). University of South Carolina Press, Columbia, SC. pp. 1-9.

- Pritchard, D. W., & Rives, S. R. (1979). Physical Hydrography and Dispersion in a Segment of the Chesapeake Bay Adjacent to the Calvert Cliffs Nuclear Power Plant. Chesapeake Bay Institute, Spec. Rept. #74, 377 pp.
- Schubel, J. R. (1971). The Estuarine Environment, American Geological Institute.
- Smith, T. J., & Takhar, H. S. (1981). A Mathematical Model for Partially Mixed Estuaries Using the Turbulence Energy Equation. <u>Estuarine</u>, Coastal and Shelf <u>Science</u>, 13(1), 27-45.
- Stanley, D. W., & Nixon, S. W. (1992). Stratification and Bottom-Water Hypoxia in the Pamlico River Estuary. <u>Estuaries</u>, <u>15(3)</u>, 270-281.
- Turner, J. S. (1973). <u>Buoyancy Effects in Fluids</u>. Cambridge Monographs on Mechanics and Applied Mathematics. Cambridge University Press. Cambridge, 368 pp.
- Uncles, R. J., & Stephens, J. A. (1996). Salt Intrusion in the Tweed Estuary. <u>Estuarine</u>, <u>Coastal and Shelf Science</u>, <u>43</u>, 271-293.
- Vieira, M. E. C. (1985). Estimates of Subtidal Volume Flux in Mid-Chesapeake Bay. Estuarine, Coastal and Shelf Science, 21, 411-427.
- Vieira, M. E. C. (1986). The Meteorologically Driven Circulation in Mid-Chesapeake Bay. <u>Journal of Marine Research</u>, <u>44</u>, 473-493.
- Wang, D. P., & Elliott, A. J. (1978). Non-Tidal Variability in the Chesapeake Bay and Potomac River: Evidence for Non-Local Forcing. <u>Journal of Physical</u>
  <u>Oceanography</u>, <u>8</u>, 225-232.
- Wang, P. F., & McCutcheon, S. C. (1993). Note on Estuary-River Models Using Boundary-Fitted Coordinates. <u>Journal of Hydraulic Engineering</u>, <u>119</u>, 1170-1175.
- Weisberg, R. H. (1976). The Non-Tidal Flow in the Providence River of Narragansett Bay: a Stochastic Approach to Estuarine Circulation. <u>Journal of Physical Oceanography</u>, <u>6</u>, 721-734.

## A. LIST OF FIGURES

1.	1995 location of the mooring	7
2.	Vertical cross section of the mouth of the Severn River with	
	the mooring Location	8
3.	1995 raw and filtered subtidal longitudinal component of the	
	current at 2.3 meters depth	10
4.	Hansen and Rattray Diagram	12
5(a).	1995 filtered air temperature time series	14
5(b).	1995 filtered water temperature time series at 2.3 and 4.7 meters depth	14
6(a).	1995 filtered temperature difference from 2.3 to 4.7 meters depth	15
6(b).	1995 filtered salinity difference from 2.3 to 4.7 meters depth	15
6(c).	1995 filtered density difference from 2.3 to 4.7 meters depth	15
7.	1995 filtered subtidal longitudinal component of the current	
	at 2.3 and 4.7 meters depth	17
8(a).	1995 Gradient Richardson Number	18
8(b).	1995 filtered longitudinal component of the wind stress	18
9(a).	1995 energy density spectrum of the wind stress	20
9(b).	1995 energy density spectrum of the current 2.3 meters depth	20

#### **B. MATHEMATICA PROGRAMS**

The following is a list of Mathematica programs written for this project. Wherever a program is reduced to a general format, it was not unique to one parameter in the actual computations. These commands were repeated many times due to the number of variables involved, i.e. temperature, salinity, and density. Programming applying to both current and wind varied only slightly and a common format for the two is given in these examples.

Comments are contained within the parentheses following certain lines to help interpret some codes. These lines can be inserted into the programming itself, as shown, without effecting the output. In addition, it is important to note that the Lanczos Filter required different numerical values for the air data and the water data. This was due to a twenty minute time interval between readings for the water instruments and a one hour time interval for air measurements. The water data filter is shown below.

## B.1 Calculating the Lanczos Filter

```
deltat = 20;
       (*sampling interval in minutes*)
Tc = 34*60;
       (*cutoff period in minutes*)
Hoprime = N[2*deltat/Tc];
       (*central weight*)
n = 356:
       (*n+1 is the length of the window*)
constant1 = N[Pi*Hoprime];
constant2 = N[2*Pi/(n+1)];
constant3 = N[constant1*constant2];
constant4 = N[Hoprime/constant3];
Hiprime = constant4*Table[N[Sin[i*constant1]*Sin[i*constant2]/i^2],{i,1,n/2}];
       (*other weights*)
denominator = Hoprime+2.0*Sum[Hiprime[[i]],{i,Length[Hiprime]}];
       (*normalization constant*)
Ho = Hoprime/denominator;
       (*normalized central weight*)
Hi = Hiprime/denominator;
       (*normalized other weights*)
part1 = Reverse[Hi];
       (*first part of the filter*)
part2 = Append[part1,Ho];
       (*combines the first part with the central weight*)
lanczos = Flatten[Append[part2,Hi]];
       (*combines the second part with the remainder of the filter*)
```

#### B.2 Filtering and Plotting Parameters

```
data = ReadList["file", Number];
        (*reads a file as a list of numbers*)
rawparameter = Table[data[[n+i]],{i,0,Length[data]-(x-n),x}];
        (*creates a list of the nth parameter in the data set*)
filtparameter = Table[Sum[rawparameter[[i+j-1]]*lanczos[[i]],{i,1,Length[lanczos]}],
        {i,1,Length[rawparameter]-(Length[lanczos]-1)}];
        (*filters the parameter using the Lanczos filter*)
newfiltparameter = Table[{i*N[1/y],filtparameter[[i]]},{i,1,Length[filtparameter]}];
       (*assigns a fraction of a day to each point depending on y readings per day*)
plotparameter = ListPlot[newfiltparameter];
        (*plots the new list - optional commands can modify or label the plot*)
Display["filtparameter.ps",plotparameter];
!psfix filtparameter.ps >! filtparameter.PS;
       (*creates a post script file of the filtered list*)
       B.3 Calculating the Longitudinal Component of the Current or Wind
component = \{Cos[2*N[Pi]/9, -Sin[2*N[Pi]/9]\};
       (*constant used to calculate longitudinal component of the current or wind*)
speed = Table[data[[n1+i]],{i,0,Length[data]-(x-n1),x}];
       (*creates a list of the current or wind velocity*)
direction = (N[Pi]/180)*Table[data[[n2+i]],{i,0,Length[data]-(x-n2),x}];
       (*creates a list of the current or wind direction in radians based on degrees true*)
vector = Table[{speed[[i]]*Cos[direction[[i]]],speed[[i]]*Sin[direction[[i]]]},
       {i,1,Length[direction]}];
       (*seperates the wind velocity into its component vectors*)
rawparameter = Table[component.vector[[i]],{,1,Length[vector]}];
       (*creates an unfiltered list of the longitudinal component*)
       B.4 Calculating Fourier Coefficients
<<filtparameter;
       (*reads the list of the filtered parameter*)
dmfiltparameter = Table[filtparameter[[i]]-Sum[filtparameter[[i]]/Length[filtparameter]],
       {i,1,Length[filtparameter]}];
       (*creates a demeaned list of the filtered parameter*)
n = Length[dmfiltparameter];
       (*length of the demeaned list*)
A[q_] := N[2/n*Sum[dmfiltparameter[[i]]*Cos[2*N[Pi]*q*i/n],{i,1,n}]];
       (*first Fourier coefficient*)
B[q_{-}] := N[2/n*Sum[dmfiltparameter[[i]]*Sin[2*N[Pi]*q*i/n], \{i,1,n\}]];
       (*second Fourier coefficient*)
```

```
aa = Table[A[q], \{q, 1, n/2\}];
       (*list of the first coefficients*)
bb = Table[B[q],\{q,1,n/2\}];
       (*list of the second coefficients*)
powparameter = aa^2+bb^2;
       (*creates a list of the squares of the coefficients, i.e. the power spectrum*)
newpowparameter = Table[\{i*N[x/n],powparameter[[i]]\},\{i,1,Length[powparameter]\}];\\
       (*assigns a frequency value to each point - x is the number of recordings per
       day*)
       B.5 Calculating the Energy Density Spectrum
n = 11:
       (*number of points to be averaged*)
number = N[1/n];
       (*averaging factor*)
smooth = Table[number[i],{i,1,n}];
       (*filter created to "smooth" the spectrum*)
<<pre><<powparameter;</pre>
       (*reads the power spectrum of the parameter*)
smooth parameter = Table[Sum[powparameter[[i+j-1]]*smooth[[i]], \{i,1,n\}],\\
       {j,1,Length[powparameter]-(n-1),n}];
       (*filters the power spectrum using the "smoothing" filter*)
       B.6 Calculating Coherence
n = 11;
number = N[1/n];
smooth = Table[number[i],{i,1,n}];
<<filtparameter1;</pre>
<<filtparameter2;
discrete1 = Fourier[filtparameter1];
discrete2 = Fourier[filtparameter2];
smoothparameter1 = Table[Sum[discrete1[[i+j-1]]*smooth[[i]],{i,1,n}],
        \{i,1,Length[discrete1]-(n-1),n\}\};
smoothparameter2 = Table[Sum[discrete2[[i+j-1]]*smooth[[i]],{i,1,n}],
        {i,1,Length[discrete2]-(n-1),n}];
       (*these first steps do the same thing as in B.6 but with complex numbers*)
product = Conjugate[discrete1]*discrete2;
       (*multiplies the complex conjugate of the first list by the second list*)
smoothproduct = Table[Sum[product[[i+j-1]]*smooth[[i]],{i,1,A}],
        {i,1,Length[product]-(A-1),A}];
       (*"smooths" this new list*)
numerator = Abs[smoothproduct];
```

(\*creates the numerator of the coherence using the absolute value of the list\*) coherence = numerator/(Abs[smoothparameter1]\*Abs[smoothparameter2]);
 (\*divides the numerator by the product of the absolute value of the two parameters\*)

## B.7 Calculating the Gradient Richardson Number

```
<<filttopdensity;
       (*filtered density data at 2.3 meter depth*)
<<filtbotdensity;
       (*filtered density data at 4.7 meter depth*)
<<filttopcurrent;
       (*filtered longitudinal current velocity data at 2.3 meter depth*)
<<filtbotcurrent;
       (*filtered longitudinal current velocity data at 4.7 meter depth*)
rho = Sum[(filttopdensity[[i]]+filtbotdensity[[i]])/(2*Length[filttopdensity]),
       {i,1,Length[filttopdensity]}];
       (*average density of the entire water column for the data set*)
deltaz = 2.4;
       (*distance between meters*)
numerator = Table[(filttopdensity[[i]]-filtbotdensity[[i]])/deltaz,
       {i,1,Length[filttopdensity]}];
       (*the numerator is the density gradient of the column*)
denominator = Table[((filttopcurrent[[i]]-filtbotcurrent[[i]])/deltaz)^2,
       {i,1,Length[filttopcurrent]}];
     (*the denominator is the velocity shear squared*)
richardson = (-9.8/rho)*(numerator/denominator);
       (*multiplies their quotient by gravity and divides by the average density*)
```